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Statistical mechanical properties of the q-oscillator system*

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Abstract. The q-deformed thermo field dynamics is constructed and in terms of this dynamics some statistical mechanical properties of a system of q-harmonic oscillators are discussed.

1. Introduction

The study of exactly solvable statistical models has led to a new algebra, which is the q-deformation of the universal enveloping algebras. These deformed algebraic structures are now usually called quantum groups [1, 2]. In the last few years, considerable interest in mathematical physics has concentrated on the quantum groups and the corresponding algebras.

Of particular interest is the boson realization of the quantum algebra in terms of the q-analogue of the quantum harmonic oscillators, which plays an important role in the study of quantum groups. Macfarlane [3], Biedenharn [4] and Sun [5] have considered a new algebra of the creation and annihilation operators of bosons which may be used to provide a realization of the quantum group $SU_q(2)$ in terms of the q-analogue of the bosonic harmonic oscillators. The q-bosonic algebra is given, in terms of a, $a^\dagger$ and N, by

$$\begin{aligned} &[a, a] = [a^\dagger, a^\dagger] = 0 \\ &aa^\dagger - qa^\dagger a = q^{-N} \\ &[N, a^\dagger] = a^\dagger \qquad [N, a] = -a. \end{aligned} \tag{1}$$

Here N is a number operator and the deformation parameter q is taken to be $q > 0$, for simplicity.

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The q-Fock space is constructed as

$$F=\left\{|n\rangle=\frac{(a^{\dagger})^n}{([n]!)^{1/2}}|0\rangle,\ a|0\rangle=0,\ n=0,1,2,\ldots\right\} \tag{2}$$

with $[n]!=[n][n-1]\ldots[2]1$. The notation $[x]$ is defined in terms of the deformation parameter q as

$$[x]=\frac{q^x-q^{-x}}{q-q^{-1}}. \tag{3}$$

The actions of $a^\dagger$, a and N on the number state $|n\rangle$ yield

$$\begin{aligned} a^\dagger|n\rangle&=[n+1]^{1/2}|n+1\rangle\\ a|n\rangle&=[n]^{1/2}|n-1\rangle\\ N|n\rangle&=n|n\rangle. \end{aligned} \tag{4}$$

In the q-Fock space, there exist the identities

$$a^\dagger a=[N] \qquad aa^\dagger=[N+1]. \tag{5}$$

One can also discuss q-deformed fermionic oscillators [6, 7]. The eigenvalues of the number operator N, for this case, can take on the values $n=0$, 1 only. The algebra of q-fermionic creation and annihilation operators is

$$\begin{aligned} &\{b,b\}=\{b^\dagger,b^\dagger\}=0\\ &bb^\dagger+q^{-1}b^\dagger b=q^{-N}\\ &[N,b^\dagger]=b^\dagger \qquad [N,b]=-b. \end{aligned} \tag{6}$$

The actions of $b^\dagger$, b and N on the state $|n\rangle$ yield

$$\begin{aligned} b^\dagger|0\rangle&=|1\rangle\\ b|0\rangle&=0=b^\dagger|1\rangle\\ N|n\rangle&=n|n\rangle \end{aligned} \tag{7}$$

with $n=0,1$, and the identities

$$b^\dagger b=[N] \qquad bb^\dagger=[1-N] \tag{8}$$

are satisfied.

For a system of q-deformed oscillators, since the usual commutation relations are replaced by the q-deformed commutation relations given above, the physical properties of the system should be affected by the deformation parameter q. The purpose of this paper is to discuss some statistical mechanical properties of the q-deformed oscillator system by using the q-analogue of the thermo field dynamics (TFD) constructed in the next section.

The paper is organized as follows. In section 2 we shall briefly review TFD [8] and also construct the q-deformed TFD (q-TFD) analogue to the usual TFD. In section 3 we shall investigate some statistical mechanical properties of a system of q-deformed harmonic oscillators by using the q-analogue of TFD. Our results are discussed and summarized at the end of the paper.

2. The *q*-analogue of TFD

Before constructing the q-TFD let us briefly review TFD. The basic idea of TFD is quite simple. As is well known, the ensemble average of an operator A is, by definition,

$$\langle A\rangle = Z^{-1}(\beta)T_r(A\,e^{-\beta H}) \tag{9}$$

where the partition function is

$$Z(\beta) = T_r\,e^{-\beta H} \tag{10}$$

and H is the Hamiltonian of the system. In quantum field theory, on the other hand, the expectation value of an operator A is given by $\langle 0|A|0\rangle$.

A natural question that arises is whether one can define a thermal vacuum $|0(\beta)\rangle$ such that the ensemble average of a quantity A can simply be written as a thermal vacuum expectation value of the operator. That is,

$$\langle 0(\beta)|A|0(\beta)\rangle = Z^{-1}(\beta)T_r(A\,e^{-\beta H}). \tag{11}$$

Equation (11) can be satisfied if one doubles the Hilbert space by introducing the tilde states $|\tilde{n}\rangle$ and defines the thermal vacuum as

$$|0(\beta)\rangle = Z^{-1/2}(\beta)\sum_n \exp(-\beta E_n/2)|n\tilde{n}\rangle \tag{12}$$

with

$$|n\tilde{n}\rangle = \begin{cases} \dfrac{(a^\dagger)^n \tilde{a}^{\dagger n}}{n!}|0\tilde{0}\rangle & n=0,1,2,\ldots \quad \text{for bosons} \\ (b^\dagger)^n(\tilde{b}^\dagger)^n|0\tilde{0}\rangle & n=0,1 \quad \text{for fermions.} \end{cases} \tag{13}$$

Here $a^\dagger$ and $b^\dagger$ are the usual (not q-deformed) creation operators of bosons and fermions, respectively, and $\tilde{a}^\dagger$ and $\tilde{b}^\dagger$ are the corresponding tilde operators. We may then directly verify equation (11) as

$$\begin{aligned}\langle 0(\beta)|A|0(\beta)\rangle &= Z^{-1}(\beta)\sum_m\sum_n \exp[-\beta(E_m+E_n)/2]\langle \tilde{m}m|A|n\tilde{n}\rangle \\ &= Z^{-1}(\beta)\sum_m\sum_n \exp[-\beta(E_m+E_n)/2]\langle m|A|n\rangle\delta_{mn} \\ &= Z^{-1}(\beta)T_r(A\,e^{-\beta H}).\end{aligned} \tag{14}$$

The advantages of this method are that all the Feyman techniques of zero-temperature field theory can be extended to this formalism. The price one has to pay is to double the degrees of freedom by introducing fictitious tilde operators corresponding to each physical operator. The tilde operators such as creation and annihilation operators satisfy the same commutation (anticommutation) relations as the physical ones and commute (anticommute) with them.

The thermal vacuum is normalized as

$$\langle 0(\beta)|0(\beta)\rangle = 1. \tag{15}$$

We now wish to construct the q-TFD analogue to the TFD reviewed above. The crucial point is to define the q-deformed thermal vacuum $|0(\beta)\rangle_q$ in a consistent way.

Define

$$|0(\beta)\rangle_q = Z^{-1/2}(\beta)\sum_n \exp(-\beta E_n/2)|n\tilde{n}\rangle_q = \sum_n (P_n)^{1/2}|n\tilde{n}\rangle_q \tag{16}$$

where $Z(\beta)$ is the partition function and

$$P_n = \frac{\exp(-\beta E_n)}{Z(\beta)} = \frac{\exp(-\beta E_n)}{\Sigma_n \exp(-\beta E_n)} \tag{17}$$

denotes the probability of a state with energy E_n. The q-Fock space is now given by

$$\left\{ |n\tilde{n}\rangle_q = \frac{(a^\dagger)^n (\tilde{a}^\dagger)^n}{[n]!} |0\tilde{0}\rangle,\ n = 0, 1, 2, \ldots \right\} \tag{18}$$

for the boson system and

$$\{ |n\tilde{n}\rangle_q = (b^\dagger)^n (\tilde{b}^\dagger)^n |0\tilde{0}\rangle,\ n = 0, 1 \} \tag{19}$$

for the fermion system. The operators $a^\dagger$ and $b^\dagger$ are now q deformed and satisfy the algebra given in equations (1) and (6), respectively. The operators $\tilde{a}^\dagger$ and $\tilde{b}^\dagger$ are the tilde operators corresponding to the operators $a^\dagger$ and $b^\dagger$. The duality of the Hilbert space requires that the tilde operators $\tilde{a}, \tilde{a}^\dagger$ and $\tilde{b}, \tilde{b}^\dagger$ satisfy the same algebra as the corresponding non-tilde operators. That is,

$$\begin{aligned} &[\tilde{a}, \tilde{a}] = [\tilde{a}^\dagger, \tilde{a}^\dagger] = 0 \\ &a\tilde{a}^\dagger - q\tilde{a}^\dagger \tilde{a} = q^{-\tilde{N}} \\ &[\tilde{N}, \tilde{a}^\dagger] = \tilde{a}^\dagger \qquad [\tilde{N}, \tilde{a}] = -\tilde{a} \end{aligned} \tag{20}$$

for bosonic operators and

$$\begin{aligned} &\{\tilde{b}, \tilde{b}\} = \{\tilde{b}^\dagger, \tilde{b}^\dagger\} = 0 \\ &\tilde{b}\tilde{b}^\dagger + q^{-1}\tilde{b}^\dagger \tilde{b} = q^{-\tilde{N}} \\ &[\tilde{N}, \tilde{b}^\dagger] = \tilde{b}^\dagger \qquad [\tilde{N}, \tilde{b}] = -\tilde{b} \end{aligned} \tag{21}$$

for fermionic operators.

The normalization condition leads to

$$\langle 0(\beta) | 0(\beta) \rangle_q = \sum_m \sum_n (P_m)^{1/2} (P_n)^{1/2} \langle \tilde{m} m | n\tilde{n} \rangle_q = \sum_m \sum_n (P_m)^{1/2} (P_n)^{1/2} \delta_{mn} = \sum_n P_n = 1. \tag{22}$$

In order to give a detailed form of the q-thermal vacua for boson and fermion systems, consider now a system of q-deformed harmonic oscillators. From the viewpoint of second quantization, the energy of the n_kth excitation $E_k = n_k \omega_k$ ($\hbar = 1$). Thus we find

$$P_{n_k} = \frac{\exp(-\beta E_k)}{\Sigma_{n_k=0}^{\infty} \exp(-n_k \beta \omega_k)} = [1 - \exp(-\beta\omega_k)] \exp(-n_k \beta \omega_k) \tag{23}$$

for bosonic oscillators and

$$P_{n_k} = \frac{\exp(-\beta E_k)}{\Sigma_{n_k=0}^{1} \exp(-n_k \beta \omega_k)} = \frac{\exp(-n_k \beta \omega_k)}{1 + \exp(-\beta \omega_k)} \tag{24}$$

for fermionic oscillators.

Substituting P_{n_k} into equation (16) we find

$$|0(\beta)\rangle_q = \sum_{n_k} (P_{n_k})^{1/2} |n_k \tilde{n}_k\rangle_q = [1 - \exp(-\beta\omega_k)]^{1/2} \sum_{n_k} \exp(-n_k \beta \omega_k / 2) |n_k \tilde{n}_k\rangle_q \tag{25}$$

for the boson system and

$$|0(\beta)\rangle_q = \sum_{n_k} (P_{n_k})^{1/2} |n_k \tilde{n}_k\rangle_q = [1+\exp(-\beta\omega_k)]^{-1/2} \sum_{n_k} \exp(-n_k\beta\omega_k/2) |n_k\tilde{n}_k\rangle_q \tag{26}$$

for the fermion system.

Having defined the q-thermal vacua we can define further q-thermal creation and annihilation operators through the Bogoliubov transformation. By the action of the q-thermal creation operators on the q-thermal vacuum, a complete orthonormal set of state vectors containing $|0(\beta)\rangle_q$ as one of the members can finally be found. This process is completely parallel to the corresponding process in TFD.

3. Statistical mechanical properties of the q-oscillator system

It is well known that the properties of a quantized physical system can be characterized by the Hamiltonian of the system and the commutation relation of the field operators or the creation and annihilation operators in the number representation. A natural question that then arises is whether a system of free harmonic oscillators is still free, or what will happen when the commutation relation is q deformed. In order to investigate this problem one can keep the free Hamiltonian of the oscillator system and impose the q-deformed commutation relation on the system. Using the above considerations we will discuss in this section some statistical mechanical properties of a system of q-deformed harmonic oscillators.

Consider now a system of q-bosonic oscillators. The free Hamiltonian of the system is chosen to be [9]

$$H = \sum_k \omega_k N_k \tag{27}$$

where ω_k is the frequency (energy) of an oscillator in the kth level and N_k is the corresponding number operator.

3.1. The q-analogue of the statistical distribution

The q-analogue of the distribution function for q-bosonic oscillators can be found as

$$f_k(q) = \langle 0(\beta)| a_k^\dagger a_k |0(\beta)\rangle_q = \langle 0(\beta)|[N_k]|0(\beta)\rangle_q . \tag{28}$$

By using q-thermal vacuum defined in the previous section we obtain

$$\begin{aligned} f_k(q) &= \sum_{m_k}\sum_{n_k} (P_{m_k})^{1/2}(P_{n_k})^{1/2} \langle \tilde{m}_k m_k |[N_k]| n_k \tilde{n}_k\rangle_q \\ &= \sum_{m_k}\sum_{n_k} (P_{m_k})^{1/2}(P_{n_k})^{1/2} \langle m_k|[N_k]|n_k\rangle_q \delta_{m_k n_k} \\ &= \sum_{n_k} [n_k] P_n . \end{aligned} \tag{29}$$

Substituting $[n_k] = (q^{n_k} - q^{-n_k})/(q-q^{-1})$ and P_{n_k} given in equation (23) into equation (29) one gets† [10]

$$\begin{aligned} f_k(q) &= \frac{1-\exp(-\beta\omega_k)}{q-q^{-1}} \sum_{n_k} \{q^{n_k}\exp(-n_k\beta\omega_k) - q^{-n_k}\exp(-n_k\beta\omega_k)\} \\ &= \frac{\exp(\beta\omega_k)-1}{[q\exp(-\beta\omega_k)-1][q^{-1}\exp(-\beta\omega_k)-1]} . \end{aligned} \tag{30}$$

† After submitting our manuscript, we became aware of the work of Altherr and Grandou [10]: their results coincide with our equations (30).

In order to clearly see the q-dependence of the distribution function, we set

$$\ln q = \gamma \qquad \text{or} \qquad q = e^{\gamma}. \tag{31}$$

Then equation (30) becomes

$$f_k(q) = \frac{\exp(\beta\omega_k) - 1}{[\exp(\beta\omega_k + \gamma) - 1][\exp(\beta\omega_k - \gamma) - 1]}. \tag{32}$$

From equation (32) we see that when $\gamma \to 0$, or $q \to 1$,

$$\lim_{\gamma \to 0} f_k(q) = \frac{1}{\exp(\beta\omega_k) - 1} = f_k \tag{33}$$

that is, the familiar Bose–Einstein distribution is recovered for the distribution $f_k(q)$.

In order to write the distribution function $f_k(q)$ (see equation (32)) in a further compact form we set

$$\begin{aligned} f_k(q) &= \frac{C_1}{\exp(\beta\omega_k + \gamma) - 1} + \frac{C_2}{\exp(\beta\omega_k - \gamma) - 1} \\ &= \frac{[C_1 \exp(-\gamma) + C_2 \exp(\gamma)] \exp(\beta\omega_k) - (C_1 + C_2)}{[\exp(\beta\omega_k + \gamma) - 1][\exp(\beta\omega_k - \gamma) - 1]} \end{aligned} \tag{34}$$

where the coefficients C_1 and C_2 are constants to be determined. Comparing equation (34) with equation (32) we find

$$\begin{aligned} &C_1 e^{-\gamma} + C_2 e^{\gamma} = 1 \\ &C_1 + C_2 = 1. \end{aligned} \tag{35}$$

The solution of equation (35) is

$$C_1 = \frac{e^{\gamma} - 1}{e^{\gamma} - e^{-\gamma}} = \frac{q - 1}{q - q^{-1}} \qquad C_2 = \frac{1 - e^{-\gamma}}{e^{\gamma} - e^{-\gamma}} = \frac{1 - q^{-1}}{q - q^{-1}}. \tag{36}$$

So we finally find the distribution function

$$f_k(q) = \frac{C_1}{\exp[\beta(\omega_k + \gamma/\beta)] - 1} + \frac{C_2}{\exp[\beta(\omega_k - \gamma/\beta)] - 1} \tag{37}$$

where C_1 and C_2 are given by equation (36).

We see from our result (equation (37)) that the deformation parameter γ appears in the distribution of q-bosonic oscillators as an exponential factor $\exp(\pm\gamma)$ due to the q-deformed commutation relation. The 'q-perturbation' leads to the energy shift $\pm\gamma/\beta$ of the harmonic oscillators. The distribution function is now divided into two parts: one with energy $\omega_k + (\gamma/\beta)$ and the other with energy $\omega_k - (\gamma/\beta)$. The requirement $q > 0$ guarantees the weights of the distribution $C_1, C_2 > 0$ and $C_1 + C_2 = 1$.

It is worth noticing that the q-dependence of the distribution function is contributed to only from those states $n \geqslant 2$, as seen from equation (29). This fact implies that the deformation parameter γ reflects, in this case, the interaction between the particles in the same state. This suggests to us that in the system of q-fermionic oscillators the factor $\exp(\pm\gamma)$ could not appear in the distribution function since $n = 0$, 1 only for the fermion system. We can verify this conclusion through direct calculation. The distribution function of q-fermionic oscillators is

$$f_k(q)=\langle 0(\beta)|b_k^\dagger b_k|0(\beta)\rangle_q=\sum_{n_k} P_{n_k}\langle n_k|b_k^\dagger b_k|n_k\rangle_q=\sum_{n_k}[n_k]P_{n_k}=P_1 \tag{38}$$

since $n=0, 1$ only. So we get, by using equation (24),

$$f_k(q)=P_1=\frac{1}{\exp(\beta\omega_k)+1}=f_k. \tag{39}$$

3.2. *The law of detailed balancing* [11]

The q-dependence is contained also in the statistical average of $[1+N_k]=a_k a_k^\dagger$ of the boson system. The statistical average of $[1+N_k]$ is

$$\langle\!\langle[1+N_k]\rangle\!\rangle=\langle 0(\beta)|[1+N_k]|0(\beta)\rangle_q=\sum_{n_k} P_{n_k}\langle n_k|[1+N_k]|n_k\rangle_q=\sum_{n_k}[n_k+1]P_{n_k} \tag{40}$$

with

$$[n_k+1]=\frac{q^{n_k+1}-q^{-(n_k+1)}}{q-q^{-1}} \tag{41}$$

where $\langle\!\langle\ldots\rangle\!\rangle$ denotes the statistical average.

Substituting equations (23) and (41) into equation (40) we obtain

$$\langle\!\langle[1+N_k]\rangle\!\rangle=\frac{\exp(\beta\omega_k)-1}{[q\exp(\beta\omega_k)-1][q^{-1}\exp(\beta\omega_k)-1]}\exp(\beta\omega_k)=\langle\!\langle[N_k]\rangle\!\rangle\exp(\beta\omega_k) \tag{42}$$

or

$$\frac{\langle\!\langle[N_k]\rangle\!\rangle}{\langle\!\langle[1+N_k]\rangle\!\rangle}=\exp(-\beta\omega_k). \tag{43}$$

When $q\to 1$, equation (43) reduces to

$$\frac{\langle\!\langle N_k\rangle\!\rangle}{\langle\!\langle 1+N_k\rangle\!\rangle}=\frac{f_k}{1+f_k}=\exp(-\beta\omega_k) \tag{44}$$

and so we are led to the law of detailed balancing.

3.3. *Black body radiation*

An assembly of photons is the simplest example of a boson system of independent particles. Photons are quanta of a radiation field, and so black body radiation can be regarded as a system of bosonic oscillators.

Consider now the q-photons which are continuously being absorbed and emitted by the walls of a cavity. The energy distribution of the black body radiation is $\omega f(q)$, where $f(q)$ is given by equation (37). The energy of a radiation field per unit volume, which is in the frequency range between ω and $\omega+d\omega$, is then given as

$$\begin{aligned} u_q(\omega,\beta)\,d\omega &= g(\omega)\omega f(q)\,d\omega \\ &= \frac{\omega^3}{C^3\pi^2}f(q)\,d\omega \end{aligned} \tag{45}$$

with the frequency spectrum

$$g(\omega)=\frac{\omega^2}{C^3\pi^2}. \tag{46}$$

So we obtain the Planck's law of radiation for the q-photon system:

$$u_q(\omega,\beta)\,\mathrm{d}\omega=\frac{\omega^3}{C^3\pi^2}\left(\frac{C_1}{\exp(\beta\omega+\gamma)-1}+\frac{C_2}{\exp(\beta\omega-\gamma)-1}\right)\mathrm{d}\omega \tag{47}$$

We see from equation (47) that when $\gamma\to 0$, $u_q(\omega,\beta)$ reduces to the usual Planck's law of radiation:

$$u(\omega,\beta)=\frac{1}{C^3\pi^2}\frac{\omega_k^3\,\mathrm{d}\omega}{\exp(\beta\omega)-1}. \tag{48}$$

The black body radiation of the q-boson system has also been discussed in [12] and [13]. In [12], the free Hamiltonian of the system has been q deformed, as well as the oscillator algebra, and so the results are different from ours.

3.4. *The specific heat* †

From equation (47) one can find the total energy of black body radiation. Note that the notation $[x]$ is invariant under the duality transformation $q\leftrightarrow q^{-1}$, i.e. $\gamma\leftrightarrow-\gamma$. So we can restrict q in the region $0<g<1$ and hence $\gamma<0$. Then the distribution function $f_k(q)$ can be written as

$$f(q)=\frac{C_1}{\exp[\beta(\omega-|\gamma|/\beta)]-1}+\frac{C_2}{\exp[\beta(\omega+|\gamma|/\beta)]-1}. \tag{49}$$

The distribution function $f(q)$ is positive defined only when $\omega>|\gamma|/\beta$. Thus the total energy is

$$\begin{aligned}u_q(\beta)&=\frac{1}{C^3\pi^2}\int_{|\gamma|/\beta}^{\infty}\left(\frac{C_1}{\exp[\beta(\omega-|\gamma|/\beta)]-1}+\frac{C_2}{\exp[\beta(\omega+|\gamma|/\beta)]-1}\right)(\omega-|\gamma|/\beta)^3\,\mathrm{d}\omega\\&=\frac{6K^4T^4}{C^3\pi^2}\left(C_1\zeta(4)+C_2\zeta(\mathrm{e}^{-2|\gamma|},4)\right)\end{aligned} \tag{50}$$

where K is the Boltzmann constant. The ζ-function and modified ζ-function are given, respectively, by

$$\begin{aligned}\zeta(4)&=\frac{\pi^4}{90}\\ \zeta(\mathrm{e}^{-2|\gamma|},4)&=\sum_{n=1}^{\infty}\frac{\mathrm{e}^{-2n|\gamma|}}{n^4}.\end{aligned} \tag{51}$$

So the specific heat per unit volume is

$$C_v(q)=\frac{\mathrm{d}u_q(T)}{\mathrm{d}T}=\frac{24K^4T^3}{C^3\pi^2}\left(C_1\zeta(4)+C_2\zeta(\mathrm{e}^{-2|\gamma|},4)\right) \tag{52}$$

which is proportional to T^3 as usual.

† Recently, we became aware of [14], in which the Debye law of the specific heat for q-bosons has been discussed.

If we assume the 'q-perturbation' is small enough, i.e. $|\gamma|$ is small, then

$$C_1\zeta(4)+C_2\zeta(e^{-2|\gamma|},4)=\tfrac{1}{2}\zeta(4)+\tfrac{1}{2}(\zeta(4)-2|\gamma|\zeta(3))=\zeta(4)-|\gamma|\zeta(3). \qquad (53)$$

So the total energy and specific heat simplify to

$$\begin{aligned} u_q(T)&=\frac{6k^4T^4}{C^3\pi^2}(\zeta(4)-|\gamma|\zeta(3)) \\ C_v(q)&=\frac{24k^4T^3}{C^3\pi^2}(\zeta(4)-|\gamma|\zeta(3)). \end{aligned} \qquad (54)$$

When $\gamma=0$, these results reduce the total energy and specific heat of per unit volume for the black body radiation [11] to

$$\begin{aligned} u(T)&=\frac{\pi^2K^4T^4}{15C^3} \\ C_v(T)&=\frac{4\pi^2K^4T^3}{15C^3} \end{aligned} \qquad (55)$$

4. Concluding remarks

The statistical mechanical properties for a system of q-harmonic oscillators have been discussed by keeping the free Hamiltonian of the system and imposing q-deformed commutation relations. The results show that the deformation parameter γ appears as an exponential factor $\exp(\pm\gamma)$ in the distribution and the energy of the free q-bosonic oscillators has been shifted $\pm(\gamma/\beta)$ due to the 'q-perturbation'. The commutation relation (1) is obviously a simple generalization of the usual commutation relation, but it is not inconceivable that such a structure could emerge as a manifestation of a dynamic symmetry.

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